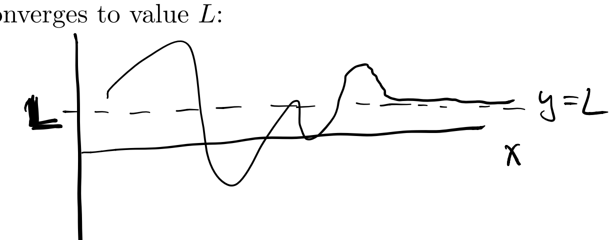
# MATH 2B Review: Limits at Infinity

## Facts to Know:

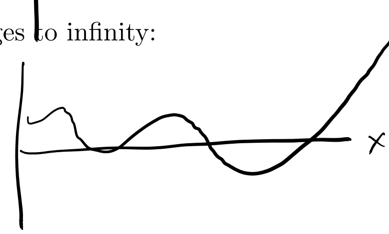
$\lim_{x\to\infty} f(x)$ describes what happens to a function as $x$ gets very large.

Finding the Limit from a Graph:

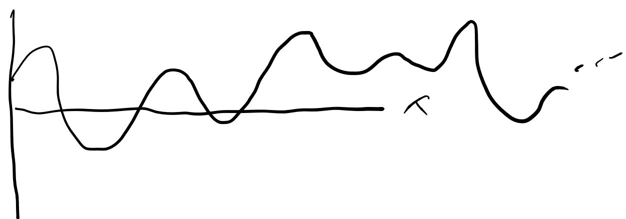
- Converges to value $L$:




- Diverges to infinity:




- Diverges:




Rational Functions:

(polynomial divided by another polynomial)

* divide numerator and denominator by the largest power of x in the fraction

Note: also works for functions with square roots

L'Hospital's Rule:

- Indeterminate Forms:

$\frac{0}{0}, \frac{\infty}{\infty}$ $\qquad \lim_{x\to\infty} \frac{f(x)}{g(x)}$

$\frac{0}{0}$ form means $\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to\infty} g(x) = 0$

- If in an indeterminate form, $\lim_{x\to\infty}$ =

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$$

Note: this isn't quotient rule

## Examples:

1. Calculate $\lim_{x\to\infty} \frac{x^2+3x}{\sqrt{4x^4-3}}$

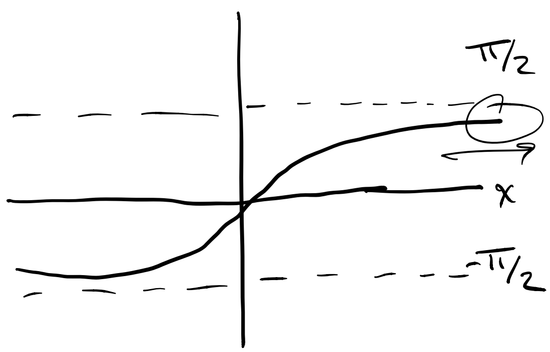
$\cancel{x^4}$ largest power ✗

$\sqrt{x^4} = x^2$ largest power ✓

$$\lim_{x\to\infty} \frac{x^2+3x}{\sqrt{4x^4-3}} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{1+\frac{3}{x}}{\sqrt{4-\frac{3}{x^4}}} = \frac{1+0}{\sqrt{4-0}} = \frac{1}{\sqrt{4}} = \boxed{\frac{1}{2}}$$

$$\frac{1}{x^2} = \frac{1}{\sqrt{x^4}}$$

2. Use the graph of $\arctan(x)$ to determine $\lim_{x\to\infty} \arctan(x)$.




$$\boxed{\lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}}$$

3. Calculate the limit $\lim_{x\to\infty} \frac{x^2}{e^x}$.

$\lim_{x\to\infty} x^2 = \infty$

$\lim_{x\to\infty} e^x = \infty$

* $\frac{\infty}{\infty}$ indeterminate form

$$\lim_{x\to\infty} \frac{x^2}{e^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2}{e^x} = \boxed{0}$$

$\lim_{x\to\infty} 2x = \infty$

$\lim_{x\to\infty} e^x = \infty$

* $\frac{\infty}{\infty}$ indeterminate form!